Hyper-frequency pulsed measurements have been performed at 3GHz. The stimulus duration was 300ns and the profile duration, which corresponds to the time for which the scattering parameters measurements are performed during the stimulus duration, was 250ns. The maximum stable gain MSG was determined under different pulsed conditions at $Vds$ = 10V (Fig. 4). At room temperature in class B for the quiescent bias point, the MSG was ~4dB. If the device was polarised in class A, the MSG obtained was 1dB better because the device is hotter under these conditions and the surface trap states were lower [6]. The MSG obtained at a chuck temperature of 150°C in class B was 2.5dB better than the first MSG for the same reason. So, by heating the device, the MSG can be improved.

*Conclusion:* Pulsed measurements have demonstrated the presence of electrical traps. The electrons can be untrapped by light and/or temperature and/or the quiescent bias point. Hyper-frequency pulsed measurements at 3GHz seem to prove that when the device is heated, the MSG improves. That is the reason why the GaN material is a good candidate for high power and high temperature applications.

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**References**

1 SULLIVAN, G.J., HIGGINS, J.A., CHEN, M.Y., YANG, J.W., CHEN, Q., PIERSON, R.L., and MCDERMOTT, B.T.: 'High power operation of AlGaN/GaN HEMTs grown on insulating silicon carbide substrates', *Electron. Lett.*, 1998, **34**, (9), pp. 922–924

2 BANDIC, Z.Z., PIQUETTE, E.C., BRIDGER, P.M., BEACH, R.A., KUECH, T.F., and MCGILL, T.C.: 'Nitride based high power devices: design and fabrication issues', *Solid-State Electron.*, 1998, **42**, (12), pp. 2289–2294

3 MORKOÇ, H., STRITE, S., GAO, G.B., LIN, M.E., SVERDLOV, B., and BURNS, M.: 'Large-band-gap SiC, III-V nitride, and II-VI ZnSe-based semiconductor device technologies', *J. Appl. Phys.*, 1994, **76**, (3), pp. 1363–1398

4 CHOW, T.P., and TYAGI, R.: 'Wide bandgap compound semiconductors for superior high-voltage unipolar power devices', *IEEE Trans.*, 1994, **ED-41**, (8), pp. 1481–1483

5 GAQUIÈRE, C., LAFONT, J.P., and CROSNIER, Y.: 'Pulsed bias/pulsed RF characterization measurement system of FET at constant intrinsic voltages', *Microw. Opt. Technol. Lett.*, 1999, **20**, (5), pp. 349–352

6 HUANG, J.C., JACKSON, G.S., SHANFIELD, S., PLATZKER, A., SALEDAS, P.K., and WEICHERT, C.: 'An AlGaAs/InGaAs pseudomorphic high electron mobility transistor (PHEMT) for X- and Ku- band power applications', *IEEE Trans.*, 1991, **MTT-41**, (5), pp. 752–759

# Stability regions estimation for mismatched uncertain variable structure systems with bounded controllers

Kuo-Kai Shyu, Yao-Wen Tsai and Chiu-Keng Lai

The problem of estimating the stability regions of mismatched uncertain variable structure systems (VSS) with bounded controllers is considered. Based on Lyapunov stability theory, the estimations of stability regions such as the practical stability region (PSR), asymptotical stability region (ASR), and exponential stability region (ESR) are discussed.

*Introduction:* In many practical variable structure systems (VSS), the boundedness of the control input should be considered because of physical constraints; the bounded control input cannot assure the global stability of the system. Hence some approaches are proposed to estimate the stability region of mismatched uncertain VSS subject to bounded controllers. In [1, 2] techniques for estimating the ASR and PSR of mismatched uncertain VSS with a bounded controller were proposed.

In this Letter, consider the following mismatched uncertain systems:

$$\dot{x} = Ax + Bu + F(t,x) \qquad (1)$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input, and the continuous function $F(t,x)$ is the uncertainty which contains both the matched part and mismatched part. It is assumed that $F$ can be decomposed as $F(t,x) = f(t,x) + Bh(t,x)$, where $f: R^+ \times R^n \to R^n$ is the mismatched part and $h: R^+ \times R^n \to R^m$ is the matched part. We denote the switching surface by $\sigma = 0$, where the switching function $\sigma = Sx$ is an $m$-state vector. The following assumptions are needed:

**A1**: There exist known non-negative constants $k_f$ and $k_u$ such that $\|f(t,x)\| \le k_f \|x\| + k_u$.

**A2**: There exist known non-negative constants $k_h$ and $k_m$ such that $\|h(t,x)\| \le k_h\|x\| + k_m$.

**A3**: The control $u$ has an upper bound $\|u\| \le k$ such that $\mu = k - k_m$ is positive.

**A4**: The pair $(A,B)$ is completely controllable; $SB$ is invertible. The notation $\|\cdot\|$ in assumptions A1–A3 denotes the Euclidean norm of $(\cdot)$.

In this Letter we address the problems of estimating the PSR, ASR, and ESR for the mismatched uncertain VSS (eqn. 1). An example is given to illustrate that the new result gives a more improved estimation of stability regions than do existing methods.

*Estimation of stability regions:* First, the results [3] are used to determine the switching surface. Consider the nominal system $\dot{x} = Ax + Bu$, and let $J = \mathrm{diag}\{\lambda_1, \lambda_2, ..., \lambda_{n-m}\}$ where the negative and real eigenvalues $\lambda_j$, $j = 1, 2, ..., n - m$, are the desired eigenvalues in the sliding mode. Let $\lambda_{max}(J)$ and $\lambda_{min}(J)$ denote the maximum and minimum eigenvalues of $J$, respectively. By assumption A4, there exist matrices $W \in R^{n\times(n-m)}$ and $N \in R^{m\times n}$ such that $[A + BN]W = WJ$. If $SW = 0$, it can be seen that $Range(W) \cap Range(B) = \{0\}$ because $SB$ is invertible. Hence $[W\ B]$ is nonsingular. $W^g$ and $B^g$ denote the generalised inverses of $W$ and $B$, respectively. Then select $S = B^g$ and a transformation matrix $M$ such that $y = Mx$ where

$$y = Mx = \begin{bmatrix} z \\ \sigma \end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix} W^g \\ S \end{bmatrix} \qquad (2)$$

with $M{-}1 = [W\ B]$, $z = W^g x$ and $\sigma = Sx$. Using the fact $W^g AW = J$, the system of eqn. 1 in the new co-ordinates is

$$\begin{aligned} \dot{y} &= MAM^{-1}y + MB(u + h) + Mf \\ &= A_0 y + A_1 y + MB(u + h) + Mf \end{aligned} \qquad (3)$$

where the matrices $A_0$ and $A_1$ are defined by

$$A_0 = \begin{bmatrix} J & 0 \\ 0 & \lambda_{min}(J)I_m \end{bmatrix} \quad \text{and}$$
$$A_1 = \begin{bmatrix} 0 & W^g AB \\ SAW & SAB - \lambda_{min}(J)I_m \end{bmatrix} \qquad (4)$$

Before estimating the stability regions, the following result is given:

*Lemma*: Select $0 < \alpha < 1$, $0 \le \beta < 1$ and $\epsilon > 0$ such that the function $g(\alpha,\beta) = 4\alpha^2 - 4\alpha + \beta^2 + \epsilon < 0$. Define $\hat{Q} = 2(\alpha - 1)A_0$. Then the following matrix is positive definite:

$$\tilde{Q} = \begin{bmatrix} \lambda_{min}(\hat{Q}) & -q_1 & -q_2 \\ -q_1 & -2\alpha\lambda_{max}(J) & 0 \\ -q_2 & 0 & -2\alpha\lambda_{min}(J) \end{bmatrix} \qquad (5)$$

where $q_1 = \beta\lambda_{max}(J)$ and $q_2 = \sqrt{(-g(\alpha,\beta)\lambda_{min}(J)\lambda_{max}(J))}$.

*Proof of lemma*: Since $\lambda_{min}(\hat{Q}) = -2(1 - \alpha)\lambda_{max}(J) > 0$ and $g(\alpha,\beta) < 0$, we have

$$\det\begin{bmatrix} \lambda_{min}(\hat{Q}) & -q_1 \\ -q_1 & -2\alpha\lambda_{max}(J) \end{bmatrix} > \epsilon\lambda^2_{max}(J) > 0$$

Because $det(\bar{Q}) = 2\alpha\{4\alpha^2 - 4\alpha + \beta^2 - g(\alpha,\beta)\}\lambda^2_{max}(J)\lambda_{min}(J) > 0$, it is obvious that the matrix $\bar{Q}$ is positive definite.

Now, the bounded control input is defined by

$$u = -k\sigma/\|\sigma\| \qquad (6)$$

Note that this control input is bounded which satisfies assumption A3. Now, choose a Lyapunov function candidate $V = y^Ty$ and define a matrix

$$G = \begin{bmatrix} 0 & W^gAB \\ SAW & SAB/2 \end{bmatrix}$$

Since $MM^{-1} = I_n$, we have that the matrix $A_1$ defined in eqn. 4 can be rewritten by

$$A_1 = \begin{bmatrix} 0 \\ SM^{-1}G \end{bmatrix} + [0 \quad GMB] + \begin{bmatrix} 0 & 0 \\ 0 & -\lambda_{min}(J)I_m \end{bmatrix}$$

Then along any trajectory $y$ of eqn. 3, it follows from eqns. 4 and 6 that

$$\begin{aligned} \dot{V} &= -y^T\hat{Q}y + 2\alpha y^TA_0y + 2y^TA_1y \\ &\quad + 2y^TMB(u+h) + 2y^TMf \\ &= -y^T\hat{Q}y + 2\alpha z^TJz + 2\alpha\lambda_{min}(J)\sigma^T\sigma \\ &\quad + 2\sigma^TSM^{-1}Gy + 2y^TGMB\sigma - 2\lambda_{min}(J)\sigma^TSM^{-1}y \\ &\quad + 2\sigma^T(h - k\sigma/\|\sigma\|) + 2y^TMf \end{aligned} \qquad (7)$$

Use the lemma and eqn. 7, the following theorem is proposed:

*Theorem*: Let

$$\Omega_{q2} = \left\{ y : \|y\| \le \frac{\mu - k_u\|S\|}{\|H\| + k_h\|M^{-1}\| + k_f\|M\|\|B\| - q_2} \right\} \qquad (8)$$

$$\Gamma = \left\{ (z,\sigma) : \|\sigma\| = 0, \|z\| \le \frac{k_u\|W^g\|}{-k_f\|M\|\|W\| + q_1} \right\} \qquad (9)$$

where $\mu - k_u\|S\| > 0$, $-k_f\|M\|\,\|W\| + q_1 > 0$ and $H = SM^{-1}G + (GMB)^T - \lambda_{min}(J)SM^{-1}$. Then any trajectory starting in $\Omega_{q2}$ will converge to $\Gamma$ and stay in $\Gamma$ thereafter. That is, the region $\Omega_{q2}$ is an estimation of PSR with respect to $\Gamma$.

*Proof of theorem*:
($a$) $\|z\| \ne 0$ and $\|\sigma\| \ne 0$: From the fact that $x = M^{-1}y = Wz + B\sigma$, $y^TM = z^TW^g + \sigma^TS$, $y^T\hat{Q}y \ge \lambda_{min}(\hat{Q})\|y\|^2$, $z^TJz \le \lambda_{max}(J)\|z\|^2$ and using eqn. 7 and assumptions A1 and A2, we have

$$\begin{aligned} \dot{V} &\le -y^T\hat{Q}y + 2\alpha z^TJz + 2\alpha\lambda_{min}(J)\sigma^T\sigma + 2k_h\|\sigma\|\|x\| \\ &\quad + 2\|\sigma\|(-k + k_m) + 2\|\sigma\|\|y\|\{\|SM^{-1}G + (GMB)^T \\ &\quad - \lambda_{min}(J)SM^{-1}\|\} + 2k_f\|M\|\|W\|\|z\|\|y\| \\ &\quad + 2k_f\|M\|\|B\|\|\sigma\|\|y\| + 2k_u\|W^g\|\|z\| + 2k_u\|S\|\|\sigma\| \\ &\le -\lambda_{min}(\hat{Q})\|y\|^2 + 2\alpha\lambda_{max}(J)\|z\|^2 + 2\alpha\lambda_{min}(J)\|\sigma\|^2 \\ &\quad + 2\{\|SM^{-1}G + (GMB)^T - \lambda_{min}(J)SM^{-1}\| \\ &\quad + k_h\|M^{-1}\| + k_f\|M\|\|B\|\}\|\sigma\|\|y\| \\ &\quad + 2k_f\|M\|\|W\|\|z\|\|y\| \\ &\quad + 2k_u\|W^g\|\|z\| + 2(-\mu + k_u\|S\|)\|\sigma\| \end{aligned} \qquad (10)$$

Since $\|z\| \le \|y\|$, using the lemma and letting $\bar{y} = [\|y\| \; \|z\| \; \|\sigma\|]^T$, we have

$$\begin{aligned} \dot{V} &\le -\bar{y}^T\bar{Q}\bar{y} + 2\{(k_f\|M\|\|W\| - q_1)\|z\| + k_u\|W^g\|\}\|y\| \\ &\quad + 2(-\mu + k_u\|S\|)\|\sigma\| + 2\{\|SM^{-1}G \\ &\quad + (GMB)^T - \lambda_{min}(J)SM^{-1}\| + k_h\|M^{-1}\| \\ &\quad + k_f\|M\|\|B\| - q_2\}\|\sigma\|\|y\| \end{aligned} \qquad (11)$$

Hence any trajectories starting in $\Omega_{q2}$ will converge to $\Gamma$ and cannot escape $\Gamma$ through region $\Omega_{q2} \cap \{(z,\sigma) : \|z\| \ne 0, \|\sigma\| \ne 0\}$.

($b$) $\|\sigma\| = 0$ and $\|z\| \ne 0$: In this case, the purpose is to prove that any trajectory starting in $\Omega_{q2} \cap \{(z,\sigma) : \|\sigma\| = 0\}$ will converge to $\Gamma$ and stay in $\Gamma$ thereafter. From eqn. 11, it can be seen that $\dot{V} \le -\bar{y}^T\bar{Q}\,\bar{y} + 2\{(k_f\|M\|\,\|W\| - q_1)\|z\| + k_u\|W^g\|\}\|y\| \le 2\{(k_f\|M\|\,\|W\| - q_1)\|z\| + k_u\|W^g\|\}\|y\|$. Hence, any trajectory starting in $\Omega_{q2} \cap \{(z,\sigma) : \|\sigma\| = 0\}$ will converge to $\Gamma$ and stay in $\Gamma$ thereafter.

($c$) $\|z\| = 0$ and $\|\sigma\| \ne 0$: In this case, $\|y\| = \|\sigma\|$. It follows eqn. 10 and the lemma that $\dot{V} \le -\bar{y}^T\bar{Q}\,\bar{y} + 2\{-\mu + k_u\|S\|\}\,\|\sigma\| + 2\,\{\|SM^{-1}G + (GMB)^T - \lambda_{min}(J)SM^{-1}\| + k_h\|M^{-1}\| + k_f\|M\|\,\|B\| - q_2\}\|\sigma\|^2$. Hence $\dot{V} < 0$ if the trajectory starting in $\Omega_{q2}$. This ensures that any trajectory in the region $\Omega_{q2} \cap \{(z,\sigma): \|z\| = 0\}$ will asymptotically approach $\|\sigma\| = 0$. Hence the theorem is proved.

*Remark*: For the system with only matched uncertainty, i.e. $k_f = k_u = 0$, the region $\Omega_{q2}$ is an estimation of ASR.

*Corollary*: If $k_u = 0$ and $0 \le k_f \le q_1/(\|M\|\,\|W\|)$,then $\Omega_{q2}$ is an estimation of ESR.

*Proof of corollary*: Suppose that the system trajectory is in the region $\Omega_{q2}$. Using eqn. 11, we have $\dot{V} \le -\lambda_{min}(\bar{Q})\|y\|^2$. Since $V = \|y\|^2$, we obtain $\|y(t)\|^2 \le \|y(0)\|^2\exp(-\lambda_{min}(\bar{Q})t)$. Hence the corollary is proved.

*Example:* Consider a system described by eqn. 1 with

$$A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Suppose that $\|u\| < k = 20$, $\|f(t,x)\| \le 0.2\|x\|$ and $\|h(t,x)\| \le 0.3\|x\|$. If matrix $J$ is chosen to be $J = \text{diag}\{-1,-2\}$, the sliding function can be designed using $\sigma = Sx = [0\ 1\ 1]x$. We choose $\alpha = 0.5$ and $\beta = 0.715$ such that $q_1 = 0.715$ and $q_2 = 0.988$. We then see that the conditions of the corollary hold. Hence from the theorem and corollary, we have an estimated ESR $\Omega_{0.988} = \{y : \|y\| \le 6.370\}$.

Now, using the method of Hui and Żak [1], the estimation of ASR is given by $\Sigma = \{(z,\sigma) : \|\sigma\| < 1.3,\ 4.239\,\|z\| + 1.583\,\|\sigma\| \le 20\}$. Also, using theorem 2 and remark 6 from Glazos and Żak [2], the ASR is given by $\Sigma' = \{(z,\sigma) : 3.761\,\|z\| + 3.159\,\|\sigma\| < 20\}$.

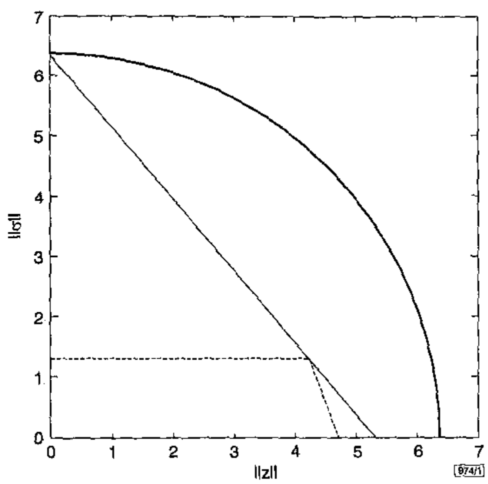


**Fig. 1** *Estimation of stability regions*

- - - - $\Sigma$
——— $\Sigma'$
——— $\Omega_{0.988}$

The stability regions $\Sigma$, $\Sigma'$ and $\Omega_{0.988}$ are shown in Fig. 1. The new result can lead to an improvement in the estimation of stability regions. It should be noted that for region $\Sigma$, by using the Hui and Żak [1] method and region $\Sigma'$ using the Glazos and Żak [2] method, only estimations of the ASR are obtained. Region $\Omega_{0.988}$ using our method is larger than $\Sigma$ and $\Sigma'$. Furthermore, $\Omega_{0.988}$ is an estimation of ESR.

 10 May 1999

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**References**

1 HUI, S., and ŻAK, S.H.: 'Robust control synthesis for uncertain/nonlinear dynamical systems', *Automatica*, 1992, **28**, pp. 289–298
2 GLAZOS, M.P., and ŻAK, S.H.: 'Practical stabilization of nonlinear/uncertain dynamic systems with bounded controllers', *Int. J. Control*, 1995, **62**, pp. 153–171
3 EL-GHEZAWI, O.M.E., ŻINOBER, A.S.I., and BILLINGS, S.A.: 'Analysis and design of variable structure systems using a geometric approach', *Int. J. Control*, 1983, **38**, pp. 657–671

# Errata

LEE, L.H.C., PERCIVAL, T.M., and SKELLERN, D.J.: 'Rate-1/2 convolutional codes over ring $Z_8$ for 8-PSK signals', *Electron. Lett.*, **35**, (11), pp. 890–892

*Editor's correction*

The expression $G_i(D) = g_{i,1} D + .... + g_{i,m} D^n$

which occurs in the sentence after eqn. 1, should read:

$G_i(D) = g_{i,0} + g_{i,1} D + .... + g_{i,m} D^n$

MONTEZUMA, P., and GUSMÃO, A.: 'Design of TC-OQAM schemes using a generalised nonlinear OQPSK-type format', *Electron. Lett.*, **35**, (11), pp. 860–861

*Editor's correction*

The expression for the modulation pulse after eqn. 6 should read:

$r_s(t) = jr(t)\exp[j(\pi t/2T)]$

CHURIN, E.G., and BAYVEL, P.: 'Passband flattening and broadening techniques for high spectral efficiency wavelength demultiplexers' *Electron. Lett.*, **35**, (1), pp. 27–28

*Author's correction*

The authors are grateful to A. Hill (BT Laboratories, Ipswich) for pointing out the inaccuracy in Figs. 1 and 2 of our Letter. The second (output) demultiplexer in these Figures should be rotated about the horizontal axis, as shown below. This inaccuracy does not affect the reported results.

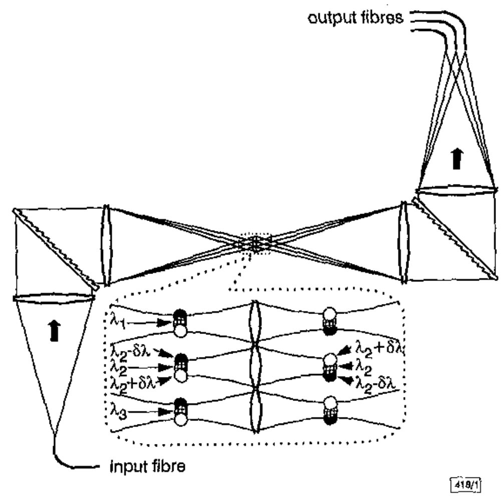


**Fig. 1** *Schematic diagram of device with two identical demultiplexers and microlens array*

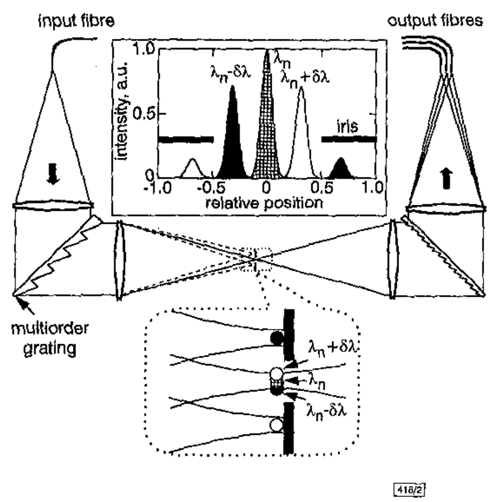


**Fig. 2** *Schematic diagram of device with multiorder diffractive element, demultiplexer, and iris*

Inset: Intensity profiles in iris plane for three wavelength channels within signal window